Multiple Reflection by Mosaic Crystals

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To optimize backscattering spectrometers with phase-space transform choppers, a mathematical description of beam deflection by mosaic crystals is needed. Analytical and numerical methods are used to improve upon the Darwin-Hamilton equations, which do not cover out-of-plane trajectories. Under realistic conditions, corrections are found to be small.

Model: Darwin's ideally imperfect crystal [1]

Crystallite distribution \( W(\theta) = G(\alpha)G(\beta) \).

Neglect primary extinction.

Relative importance of multiple-reflection contributions [4]

Lattice model (Dyck paths):

To find reflectivity by scattering order

\[ b_{n+1}(0) = C_n \left( \frac{1}{2n} \right)^{2n+1} \]

with Catalan’s numbers

\[ C_n = c_{n,n} = \frac{(2n)!}{n!(n+1)!} \]

\[ \Rightarrow \] Only a few reflections contribute substantially.

Out-of-plane multiple-reflection trajectories [5]

Generalized Darwin-Hamilton equations:

\[
\hat{\theta} \theta J_{n}(k, z) = -i(k)J_{n}(k, z) + \int d^{3}k' \mu(k, k')J_{n}(k', z)
\]

with kernel

\[ \mu(k, k') = \frac{\pi}{2} \int d\beta G(\beta)G(nk + C[I(\beta)] [1 + C[I(\beta)]]) \delta[k' - k \pm 2\pi nk(\beta)] \]

solved up to quadrature by iteration of exploded DH eqs.

\( k \) evolution independent of \( z \) ⇒ confined random walk:

Predictions [5]

Angular distributions per reflection order:

Rocking curve \( (\vartheta_k = 80^\circ, 10\% \text{ non-Bragg losses}) \):

\[ \begin{align*}
    \sigma = 1.00 & \quad \nu = 0.99 \\
    \sigma = 0.99 & \quad \nu = 0.95 \\
    \sigma = 0.96 & \quad \nu = 0.90 \\
    \sigma = 0.84 & \quad \nu = 0.82 \\
    \sigma = 0.53 & \quad \nu = 0.46
\end{align*} \]

Literature